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Assessment of maximum concentration at ground level of gas from a heated elevated source

by

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1. Introduction. The assessment of the maximum concentration at ground level of gas discharged from an elevated source, such as a tall chimney, is a matter of considerable practical importance. Many industrial chimney stacks discharge gases which are harmful to health. The gas concentration at ground level is influenced by meteorological factors and also by factors related to the stack and furnace design and it is obviously desirable that the same formula should be used in all cases for computing the maximum gas concentration likely to be experienced at ground level. The effect of a specified gas concentration varies with the individual and for this reason great accuracy in predicting the maximum concentration is not essential. The purpose of the present note is to compare three current formulae.

If the effluent has the same density as the ambient air and is discharged without significant vertical velocity there is no real difficulty. Sutton (1) has given a formula which can be used to compute ground level concentrations at various distances from the stack. If the gas experiences significant vertical movement after leaving the stack the concentration at ground level is more difficult to predict. In general the gas will be discharged into a horizontal wind and the vertical velocity of the gas will diminish as the gas gets further from the stack. The gas plume will thus be vertical at the stack exit and will then bend over in the direction of the wind and at a considerable distance from the stack the plume will be nearly horizontal. An approximate method of computing ground level concentration is thus to treat the source as a cold source emitting with zero vertical velocity but to assume that the source is raised by an amount z equal to the height of the plume above the source at a distance where the plume is "practically" horizontal. It is in the method for computing this height z that the three formulae discussed below differ.

2. Ground level concentration from a cold elevated source. Sutton (1) has shown that if the effluent has the same density as the atmosphere and is emitted without significant vertical motion the concentration at ground level increases with increasing distance from the source up to a critical distance and then decreases again. The maximum concentration at ground level is given by

$$X = \frac{2Q}{e\pi u h^2} \left(\frac{C_z}{C_y}\right) \qquad (1)$$

and occurs at a distance x given by

$$x = \left(\frac{h^2}{C_z^2}\right)^{1/(2-n)} \qquad (2)$$

where

Q = strength of source (gm/sec)

u = wind velocity

/h =

h = height of source above ground

C_y, C_z = the usual turbulence coefficients in Sutton's diffusion theory

n = a parameter which depends upon the stability of the atmosphere. Sutton suggests that with n = 0.25 equations (1) and (2) will give satisfactory results in conditions of small temperature gradient and moderate wind.

If we assume the source is raised to a height z above the top of the stack we should replace equation (1) by

$$X = \frac{A}{u(h + z)^2} \qquad (3)$$

where A ($= \frac{2Q}{e\pi} \frac{C_z}{C_y}$) is a parameter which is independent of stack design, of u and of h. It is intuitively obvious that z will decrease as u increases and it is convenient here to consider the effect of expressing z as a power of u. Suppose we put

$$z = \frac{B}{u^p} \qquad (4)$$

then X, which is already a maximum concentration with respect to distance from the stack, will be a function of u. It is easily seen that X has a maximum value with respect to u given by

$$X_{max} = \frac{A}{B^{1/p}} \quad \frac{1}{h^{2-1/p}} \quad \frac{(2p - 1)^{2-1/p}}{4p^2} \qquad (5)$$

when

$$u^p = \frac{B(2p - 1)}{h} \qquad (6)$$

$$z = \frac{h}{2p - 1} \qquad (7)$$

The distance at which this maximum concentration occurs would be of interest in a specific case but is not relevant to the general question of stack design.

3. Sutton's treatment of a heated elevated source

Sutton considers first a steady continuous point source of heat of strength Q_H (cal/sec) discharging into a completely calm atmosphere. The plume of course is vertical. He makes the following assumptions:-

(i) The potential temperature of the ambient atmosphere is uniform at all heights and the difference between the temperature in the plume and the ambient temperature is so small that density differences may be neglected.

(ii) There is no significant variation of density with height.

(iii) The rate of entrainment of ambient air into the jet (per unit area of the jet boundary surface) is proportional to the radius of the jet and the vertical gradient of the vertical velocity.

(iv) The radius of a (circular) cross-section of the jet at any height is proportional to a power of the height, i.e. $r = 1.5\, C z^{m/2}$ where C is a generalized diffusion coefficient and m is unspecified at this stage.

Then by utilizing the principles of conservation of mass and of heat and equating the rate of dissipation of energy owing to turbulence to the rate of loss of potential energy as the air in the jet moves upwards Sutton obtains the following equation for the speed in the vertical plume

$$v = v_1 z^{-m/6} \qquad (8)$$

/where

where

$$v_1^3 = \frac{7 Q_H g}{3 \pi c_p \rho T_a C} \qquad (9)$$

c_p is the specified heat of air, ρ the density of air and Ta the air temperature. By analogy with the rate of spread of a horizontal plume from a cold source in a steady wind Sutton then deduces that m = 1.75 so that

$$v = v_1 z^{-0.29} \qquad (10)$$

To apply these results to the practical case of a hot source emitting into a steady horizontal wind of speed u Sutton then assumes that, as a first approximation, the centre line of the plume will follow the trajectory of a particle subjected to a constant horizontal velocity u and a vertical velocity v which decays according to

$$v = v_1 s^{-1/3} \qquad (11)$$

s being the distance measured along the trajectory and the index - 1/3 being taken as an approximation to - 0.29 in order to obtain explicit equations for the trajectory. Sutton describes the trajectory by the equations

$$s = \left(\frac{v_1}{u}\right)^3 \cot^3 \psi \qquad (12)$$

$$z = \frac{3}{2}\left(\frac{v_1}{u}\right)^3 F \qquad (13)$$

$$F = \cot \psi \operatorname{cosec} \psi - \ln(\cot \psi + \operatorname{cosec} \psi) \qquad (14)$$

where ψ is the inclination of the trajectory to the horizontal. When ψ is small the plume is practically horizontal and Sutton suggests that z be computed by assuming $\psi = 10^\circ$ (this gives F = 30.3). We note that equation (13) has the same form as equation (4) and that in applying Sutton's treatment to equation (5) we put

$$B = \frac{3}{2} v_1^3 F \qquad p = 3 \qquad (15)$$

It is convenient to note here that the horizontal distance corresponding to equations (12) to (14) is given by

$$x = \left(\frac{v_1}{u}\right)^3 (\operatorname{cosec}^3 \psi + 2 - 3 \operatorname{cosec} \psi) \qquad (16)$$

4. <u>The method of Bosanquet, Carey and Halton (2)</u>. These three authors have considered the rise of a plume of gas which emerges from the chimney stack with a temperature above that of the ambient air and with a definite issuing velocity. The detailed arguments by which they obtain their final formulae are not given in the published paper but are contained in a paper deposited in the library of the Institution of Mechanical Engineers. In brief their argument is as follows.

As the plume emerges (vertically) from the chimney it has a velocity relative to the ambient air. After emergence the ambient air mixes with the plume thereby reducing the excess temperature in the plume and also the relative velocity. The reduction of the excess temperature affects the buoyancy of the heated plume and so determines the upward motion of the gas. The governing factor is thus the rate of dilution of the plume by the ambient air. The authors attribute this dilution to three independent causes, viz. diffusion of air into the plume by eddy motion, longitudinal motion of the plume relative to the ambient air and transverse motion of the plume relative to the ambient air, and for each cause a "rate of inflow" of ambient air into the plume is calculated.

/From

From analogy with pipe flow and kinetic theory the authors assume that the diffusion coefficient which expresses the diffusion of air into the plume is given by $p \sigma u$ where p is an undetermined multiplier and σ is the r.m.s. distance of the particles in the plume above and below a horizontal plane through the axis. With the assumption of a conical distribution of gas density in a cross-section of the plume (the precise form of the density distribution does not seem to be important) it is then deduced that the effects of diffusion are equivalent to a radial inflow of air into the plume equal to 2.58 pu. From consideration of a paper by Heywood (3) the authors conclude that a reasonable value for p is 0.05 so that the radial inflow of air into the plume owing to eddy diffusion is 0.129u feet per second per square foot of surface where u is the wind speed in feet per second.

To deal with the effect of the motion of the plume relative to the air and parallel to the axis of the plume the authors appeal to laboratory results which show that when a jet of air issues from a cylindrical nozzle into stagnant air

(i) the jet spreads into a cone of semi-angle θ where $\tan \theta = 0.214$

(ii) the velocity distribution over all cross-sections is similar and approximates to the conical form

(iii) the mean velocity over a cross-section varies inversely as the cone radius.

From these laboratory results and with the assumption of constancy of momentum it is shown that the radial inflow into the plume is $0.107 \bar{V}$ where $\bar{V}$ is the mean relative velocity parallel to the axis over a cross-section of the plume.

The effect of the relative motion of the plume transverse to the axis is dealt with by considering the resistance to transverse motion of a solid cylinder of unit length. This is $\frac{1}{2} \rho V_T^2$ per unit projected area (ρ is the air density). If the solid cylinder is replaced by a cylinder of air the latter will experience the same resistance and will slow down but the decreasing velocity now results from the cylinder being diluted with ambient air which shares the original momentum of the cylinder. From these considerations the authors show that the radial inflow owing to transverse relative motion is $V_T/2\pi$.

Having obtained the rates of inflow arising from the three causes the total rate of inflow is assumed to be the root mean square of the sum. The rate of dilution of the plume by the ambient air is thus determined and thence it is a matter of mathematics to compute the rate of rise of the gas through the atmosphere.

The results obtained by Bosanquet, Carey and Halton are expressed in the following formulae.

Rise caused by exit velocity of stack gases

$$z_{max} = \left[\frac{4.77}{1 + 0.43\frac{u}{v}}\right] \left[\frac{\sqrt{Q_g v}}{u}\right] \qquad (17)$$

$$z = z_{max} \left(1 - 0.8 \frac{z_{max}}{x}\right) \text{ when } x > 2z_{max} \qquad (18)$$

Rise caused by density difference between stack gases and atmosphere

$$x = 3.57 \frac{\sqrt{Q_g v}}{u} X \qquad (19)$$

$$z = 6.37 g \frac{Q_g \Delta}{u^3 T_1} Z \qquad (20)$$

provided $u^2 > \frac{\Delta g}{T_1} \sqrt{\frac{Q_g}{v}}$ and where X and Z are related by the curve given in figure 2.

/Maximum

Maximum rise of plume above stack exit

The plume reaches a maximum height when its density becomes equal to that of the surrounding atmosphere and when the potential atmospheric temperature gradient is positive. The maximum rise owing to the exit velocity is given by equation (17). The maximum rise owing to density difference between the issuing gas and the atmosphere is given by

$$z_{max} = 6.37 g \frac{Q_g \Delta}{u^3 T_1} (\ln J^2 + \frac{2}{J} - 2) \qquad (21)$$

where

$$J = \frac{u^2}{Q_g v} \left[0.43 \sqrt{\frac{T_1}{gG}} - 0.28 \frac{v}{g} \frac{T_1}{\Delta} \right] + 1 \qquad (22)$$

In these formulae

Q_g = rate of exit of gas from stack in $ft^3 sec^{-1}$, the gas being assumed to have a temperature at which its density equals atmospheric density

v = actual gas velocity at stack exit ($ft\ sec^{-1}$)

T_1 = temperature at which density of flue gases equals that of the atmosphere (^oC abs.)

Δ = temperature difference between actual flue gas temperature and T_1 (^oC)

G = gradient of potential atmospheric temperature ($^oC\ ft^{-1}$).

5. The Oak Ridge formula. This is a purely empirical formula derived from observations of the height at which the plumes from three different stacks at Oak Ridge (Tennessee, U.S.A.) became horizontal. The formula arose from a suggestion that $uz = 1.5vd$ for each stack (u = wind speed (m.p.h.), z = height above stack exit (ft), v = stack gas exit velocity (m.p.h.) and d = stack diameter (ft)). On test it became clear that this simple formula is inadequate and, in fact, it contains no reference to the temperature of the issuing gas. The authors of the report then plotted $(uz - 1.5vd)$ against Q_H and obtained the curve shown in figure 1. From this figure it is clear that a reasonably accurate approximation is given by

$$uz = 1.5vd + 3 \times 10^{-4} Q_H \qquad (23)$$

It is to be noted that the X-10P point is based upon 11 observations, the X-10SP point upon 34 observations and the WBSP point upon 93 observations. The Oak Ridge report also states that in lapse conditions uz is larger than indicated by equation (23) by 10 to 20% and smaller by a similar amount in inversion conditions. If the variables in equation (23) are measured in c.g.s. units (Q_H in cal/sec) the equation becomes

$$uz = 1.5vd + 0.4\, Q_H \qquad (24)$$

It will be noted that this equation is equivalent to equation (4) if we put

$$B = 1.5vd + 0.4\, Q_H \qquad p = 1 \qquad (25)$$

6. Comparison of the three formulae for rise of plume. Comparison of the three formulae for z is not quite straightforward since they do not contain the same parameters. Sutton's formula takes no account separately of the stack diameter, the gas temperature or the exit velocity from the stack though a combination of these three is implicit in the use of Q_H. The formula of Bosanquet et al. takes account of almost every physical variable which can affect the problem and, as a result, is somewhat cumbersome. It does not

/contain

contain Q_H explicitly but if the effluent has the physical properties of air it is clear that

$$Q_H = Q_g \rho c_p \Delta = \frac{\pi}{4} d^2 v \frac{T_1}{T_1 + \Delta} \rho c_p \Delta \qquad (26)$$

The Oak Ridge formula ignores gas temperature except insofar as it is implicit in Q_H.

In order to compare the three formulae z has been calculated for four different values of Q_H and three different values of u by each method. It was necessary to make certain assumptions about the emission. These were

(i) The exit velocity from the stack is 50 ft/sec

(ii) The effluent has the physical properties of air

(iii) The ambient and effluent temperatures are 288°A and 423°A respectively.

The rise of plume (z) computed by the three methods is given in table 1. The stack diameter d tabulated results from the assumptions listed above and equation (26).

The outstanding feature of this table is the great disparity between the values predicted by the three formulae. It must be borne in mind however that our primary interest is not in the rise of the plume but in the maximum gas concentration at ground level.

7. The maximum gas concentration at ground level. We have seen that there is an easy method of computing X_{max} if z can be represented by an equation similar to equation (4) and that in fact Sutton's formula and the Oak Ridge formula are of this form. Accordingly the total (buoyancy plus velocity) rise by the formula of Bosanquet et al. was computed for four sets of "stack conditions" and various values of u. This total rise has been plotted logarithmically against u (foot second units were employed as in the original formula) in figure 3. It is apparent that over the range u = 5 to 40 ft/sec for $Q_H = 10^5$ and over the range u = 20 to 150 ft/sec for $Q_H = 10^8$ can be represented by an equation similar to equation (4). It is necessary for the present argument that the critical velocity (equation (6)) shall fall within the range of u for which the power law for z is assumed. The conditions assumed in computing figure 3, the deduced values of B, p and d (c.g.s. units) and of the critical velocity (ft/sec) for a stack height of 100m are as follows.

In each case $T_1 = 288°C$ $\Delta = 135°C$

Q_H (cal/sec)	10^5	10^5	10^8	10^5
G (°C/1,000ft)	1	1	1	5
v (m/sec)	15.25	5.0	15.25	15.25
B (c.g.s.)	5.5×10^7	6.0×10^8	3.6×10^{10}	3.3×10^7
p	1.75	2.18	2.0	1.69
d (metres)	0.55	0.97	17.5	0.55
crit. vel. (ft/sec)	7.6	8.9	10.7	6.6

With z represented by equation (4) for all three formulae we now have a ready means of comparing the predicted maximum concentrations at ground level. When using Sutton's formula we put $c_p = 0.25$, $\rho = 1{\cdot}2\times10^{-3}$, $T_a = 288$ and $C = 0.3$ in equation (9) and equation (13) (with $F = 30.3$) then gives $z = 1{\cdot}28\times10^6 Q_H/u^3$. For the Oak Ridge formula we use $z = (1.5vd + 0.4Q_H)/u$.

/If

Table 1

Rise of plume predicted by the three formulae

Strength of heat source Q_H cal/sec		2.5×10^6			5×10^6			2.5×10^7			5×10^7		
Power station capacity M.W. (approx)		50			100			500			1,000		
Stack diameter (ft)		9.1			12.8			28.7			40.5		
Wind speed m.p.h.		6.7	10	13.3	6.7	10	13.3	6.7	10	13.3	6.7	10	13.3
Rise of plume (ft) by Sutton's formula (To 10^o inclination)		3,700	1,100	460	7,400	2,200	920	37,000	11,000	4,600	74,000	22,000	9,200
Rise of plume (ft) according to Bosanquet et al. $G = 1^oC/1,000$ ft	Buoyancy	730	310	160	1,190	540	300	3,340	1,750	1,000	4,470	2,820	1,680
	Velocity	150	95	70	210	135	99	470	300	220	667	426	312
Rise of plume (ft) according to Bosanquet et al. $G = 0.5^oC/1,000$ ft)	Buoyancy	-	-	-	-	-	-	4,420	2,120	1,180	6,700	3,550	2,020
Rise of plume (ft) by Oak Ridge formula	Buoyancy	113	75	57	230	150	113	1,125	750	570	2,250	1,500	1,130
	Velocity	68	45	34	96	64	48	214	143	108	302	203	152

If we use X_S, X_B and X_R to denote the maximum concentrations at ground level according to Sutton, Bosanquet et al. and Oak Ridge respectively then table 2 gives the ratio X_R/X_S for a variety of conditions. Table 3 gives the values of X_S/A and X_B/A for various conditions, this different form of presentation being preferred in order to emphasize some of the conditions attached to the calculations.

Table 2
Ratio X_R/X_S

v	d	h	Q_H (cal/sec)			
m/sec	m	m	10^8	10^7	10^6	10^5
5	5	50	0.23	0.97	2.57	2.22
	10		0.23	0.90	1.73	1.17
15	5		0.22	0.83	1.30	0.79
	10		0.22	0.68	0.75	0.40
45	5		0.21	0.58	0.53	0.27
	10		0.20	0.40	0.28	0.13
5	5	100	0.36	1.54	4.06	3.51
	10		0.36	1.42	2.74	1.84
15	5		0.35	1.31	2.06	1.25
	10		0.35	1.08	1.18	0.63
45	5		0.33	0.91	0.83	0.43
	10		0.31	0.63	0.44	0.22
5	5	200	0.57	2.46	6.44	5.57
	10		0.57	2.26	4.34	2.92
15	5		0.56	2.09	3.28	1.98
	10		0.55	1.72	1.88	1.01
45	5		0.53	1.45	1.32	0.67
	10		0.49	1.00	0.70	0.34

Table 3
Values of $10^{13}X_B/A$ and $10^{13}X_S/A$

G	v	d	h	Q_H (cal/sec)			
°C/1,000ft	m/sec	m	m	10^8		10^5	
				$10^{13}X_B/A$	$10^{13}X_S/A$	$10^{13}X_B/A$	$10^{13}X_S/A$
1	5	0.97	50	-	-	640	556
1	15.2	0.55	50	-	-	585	556
1	15.2	17.5	50	48.5	55.6	-	-
5	15.2	0.55	50	-	-	644	556
1	5	0.97	100	-	-	219	176
1	15.2	0.55	100	-	-	222	176
1	15.2	17.5	100	17.2	17.6	-	-
5	15.2	0.55	100	-	-	243	176
1	5	0.97	200	-	-	75.1	55.6
1	15.2	0.55	200	-	-	82.6	55.6
1	15.2	17.5	200	6.1	5.56	-	-
5	15.2	0.55	200	-	-	92.4	55.6

Table 2 shows that X_R and X_S differ by a factor exceeding 3 in only 17 out of 72 cases. To simplify discussion let us assume temporarily that X_S is "correct". Then we see that, broadly, X_R underestimates X_{max} with either very large values of Q_H or very large values of v. Now when Q_H is very large X_{max} will normally be small and the underestimation is perhaps unimportant. The largest value of v dealt with in table 2 is probably unrealistic and we need not pay undue attention to the ratios for v = 45m/sec. We have not so far considered whether the critical wind velocities corresponding to X_R and X_S in table 2 are within

/normal

normal experience. If the cases for $Q_H = 10^8$ or v = 45m/sec are excluded the greatest value of B by the Oak Ridge formula is $6{\cdot}25 \times 10^6$. For a stack 50m high this would lead to a critical wind speed of 12.5 m/sec - a value well within common meteorological experience. According to Sutton's formula $B = 1{\cdot}28 \times 10^6 Q_H$. Putting $Q_H = 10^7$ cal/sec and h = 50 metres leads to a critical wind speed of 23.3 m/sec. This is rather high but is not outside experience. The critical wind speed is lower for taller stacks or smaller values for Q_H of course.

Turning now to table 3 the only worthwhile comment seems to be one of surprise that two such dissimilar formulae should give such concordant results.

8. Conclusion. In making a selection between the three methods of computing X_{max} from a hot source the following considerations seem relevant

(i) Sutton's formula for computing concentration (equation (1)) is based upon a great deal of experimental field work. His formula for the rise of a heated plume (equation (13)) is admittedly only a first approximation and has not been compared with full scale observations (but see appendix).

(ii) The formula of Bosanquet et al. (equations (21) and (22)) has given reasonable agreement with some full scale observations. The deviation however is partly dependent upon laboratory observations of the flow from a cylindrical nozzle and the resistance of a circular cylinder in a moving fluid.

(iii) The Oak Ridge formula for the rise of the plume is empirical and expresses the result of a number of full scale observations.

(iv) The differences between the results given by the three formulae are small compared with the uncertainty and variability of the effects of specified concentrations of gas on a human being. In those conditions in which the discrepancy is largest and the concentration itself is also large the Oak Ridge formula generally gives the highest value. In view of these considerations, it is recommended that the Oak Ridge formula, combined with Sutton's equation (1), be employed for computing the maximum concentration likely to be experienced at ground level from a heated elevated source. If the discrepancy between X_R and X_S for large values of Q_H is considered important the three formulae could be brought more into line by increasing the coefficient of Q_H in equation (24) to, say, 0.8. This would have little effect on table 2 at the small values of Q_H but would improve the agreement at large values. Such an "adjustment" could be regarded only as a matter of expediency however.

/Table 4

Table 4

Rise of plume (ft) at various distances from stack as observed (z) and as predicted by Bosanquet et al. (z_B) and by Sutton (z_S)

Distance from stack (feet)

Test No.	u ft/sec		100	150	200	300	400	600	800
	Plant No. 1 Q_g = 1,000 ft^3/sec v = 31ft/sec Δ = 180°C Q_H = 1.53x10^6 cal/sec								
1	14	z	60		99		156	186	
2		z_B	77		122		176	210	
		z_S	79		131		198	265	
2	33	z	29		35		52	70	84
		z_B	22.2		27.4		32.2	35.1	37.2
		z_S	35		59		90	118	143
3	26.5	z	28		40		63	80	101
		z_B	30.6		39.8		48.9	54.3	58.6
		z_S	42		69		110	147	177
4	19	z	55		75		121	165	200
		z_B	49.9		71.9		95.2	109.4	119.3
		z_S	58		94		150	199	243
	Plant No.2 Q_g = 341 ft^3/sec v = 28ft/sec Δ = 25°C Q_H = 7.24x10^4 cal/sec								
1	10	z	46	53	58	67			
		z_B	34.6	41.9	46.3	52.3			
		z_S	41	54	66	87			
2	13.3	z	36	38	41	47			
		z_B	26.4	30	32.3	35.2			
		z_S	31	41	51	66			
	Plant No. 3 Q_g = 1,245 ft^3/sec v = 35.5 ft/sec Δ = 149°C Q_H = 1.58x10^6 cal/sec								
1	20.4	z	46		66	82	92	103	
		z_B	45.0		64.9	76.9	85.4	97.7	
		z_S	54		88	117	143	190	

/Appendix

Appendix

In the appendix to their paper Bosanquet et al. quote the rise of the plume at various distances from the stacks of four different plants and with various wind speeds. They compare these observed rises with the rise predicted by their formula. Particulars of eight such tests are given. The rise of the plume referred to here is not the rise with which we have been concerned above since all the measurements were made so close to the stack that even at the greatest distance the height of the plume was still obviously increasing. The characteristics quoted for the plants are, naturally, those used in the theoretical part of their paper. In seven of the eight cases however T_1, the temperature at which the density of the flue gases equals that of the atmosphere, is given as 283°A. This is so near to normal atmospheric temperature that it seemed justifiable to regard the effluent as air with temperature $(283 + \Delta)$°A discharging into an atmosphere at temperature 283°A. Another parameter quoted for the tests was Q_g, the volume rate of discharge at atmospheric density (in cubic feet per second). Assuming air density to be 1.2gm/cm^3 and the specific heat to be 0.25 cal/gm we then get

$$Q_H = 8.5\, Q_g\, \Delta \text{ cal/sec}$$

for the strength of heat source above atmospheric temperature. We now have the necessary data with which to compare Sutton's formula for rise of plume with the observations quoted by Bosanquet et al. From equations(13) and (16) we have

$$\frac{x}{z} = \frac{2}{3}\,\frac{(\operatorname{cosec}^3\psi + 2 - 3\operatorname{cosec}\psi)}{[\cot\psi\,\operatorname{cosec}\psi - \ln(\cot\psi + \operatorname{cosec}\psi)]}$$

This ratio is a function only of ψ. The ratio xu^3/v_1^3 is also a function only of ψ. These two ratios were computed for various values of ψ and a subsidiary curve of x/z against xu^3/v_1^3 plotted.

If in equation (9) we substitute $g = 32$, $c_p = 0.25$, $\rho = 1{\cdot}2\times10^{-3}$, $T_a = 283$ and $C = 0.3$ we obtain

$$v_1^3 = 2.86 \times 10^4\, Q_H$$

Since the values of u are quoted in each test we can thus compute xu^3/v_1^3 for any specified distance, read off x/z from the subsidiary curve, and hence compute z. In table 4 the observed rise of plume above the stack exit is denoted by z, the total rise predicted by Bosanquet et al. by z_B and the rise computed as indicated above by z_S. The paper by Bosanquet et al. does not say how the rise of plume was measured but some of the values of z are so small that one may expect the error of measurement to have been, sometimes, a significant fraction of z. For this reason it has seemed preferable to tabulate z, z_B and z_S rather than ratios. It is clear that z_S is invariably greater than either z or z_B though in view of the need for estimating some of the quantities the agreement between z and z_S may be regarded as reasonably satisfactory. On the average z_S exceeds z by a factor of about 1.4.

References

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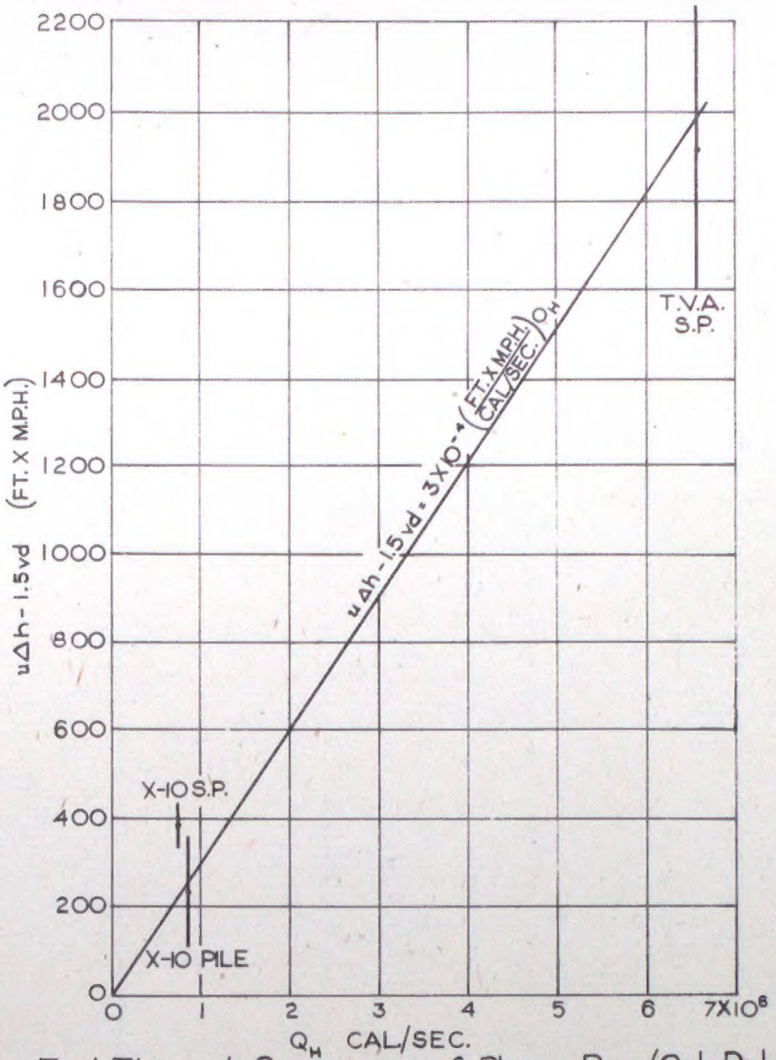

Fig.1 Thermal Component of Plume Rise (Oak Ridge)

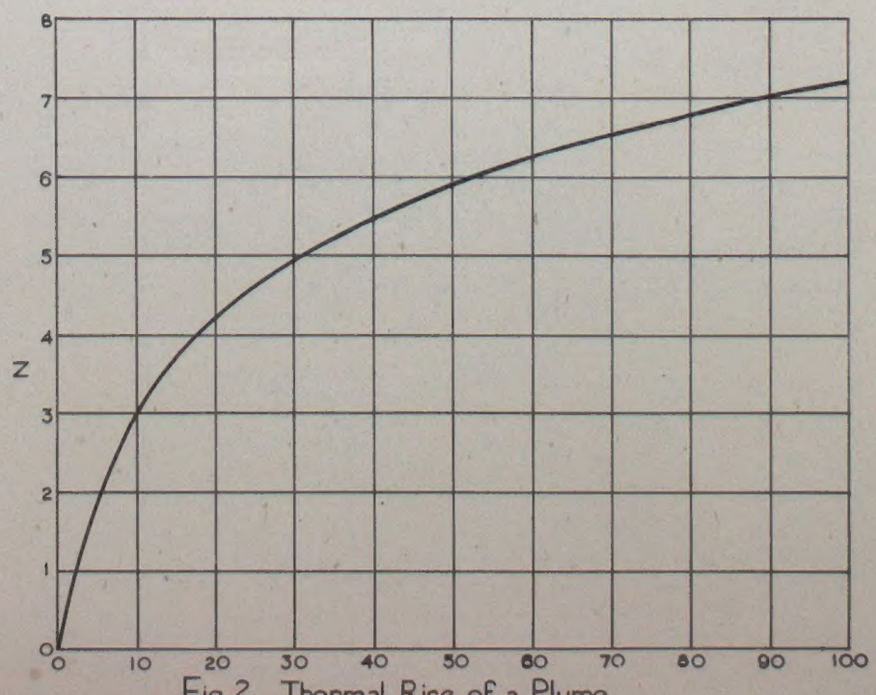

Fig.2. Thermal Rise of a Plume

Showing relation between X and Z.

$x = 3{\cdot}57 \frac{\sqrt{g \frac{\Delta T}{T}}}{u} X$ and $z = 6{\cdot}37 g \frac{Q_A}{u^3 T_1} Z$

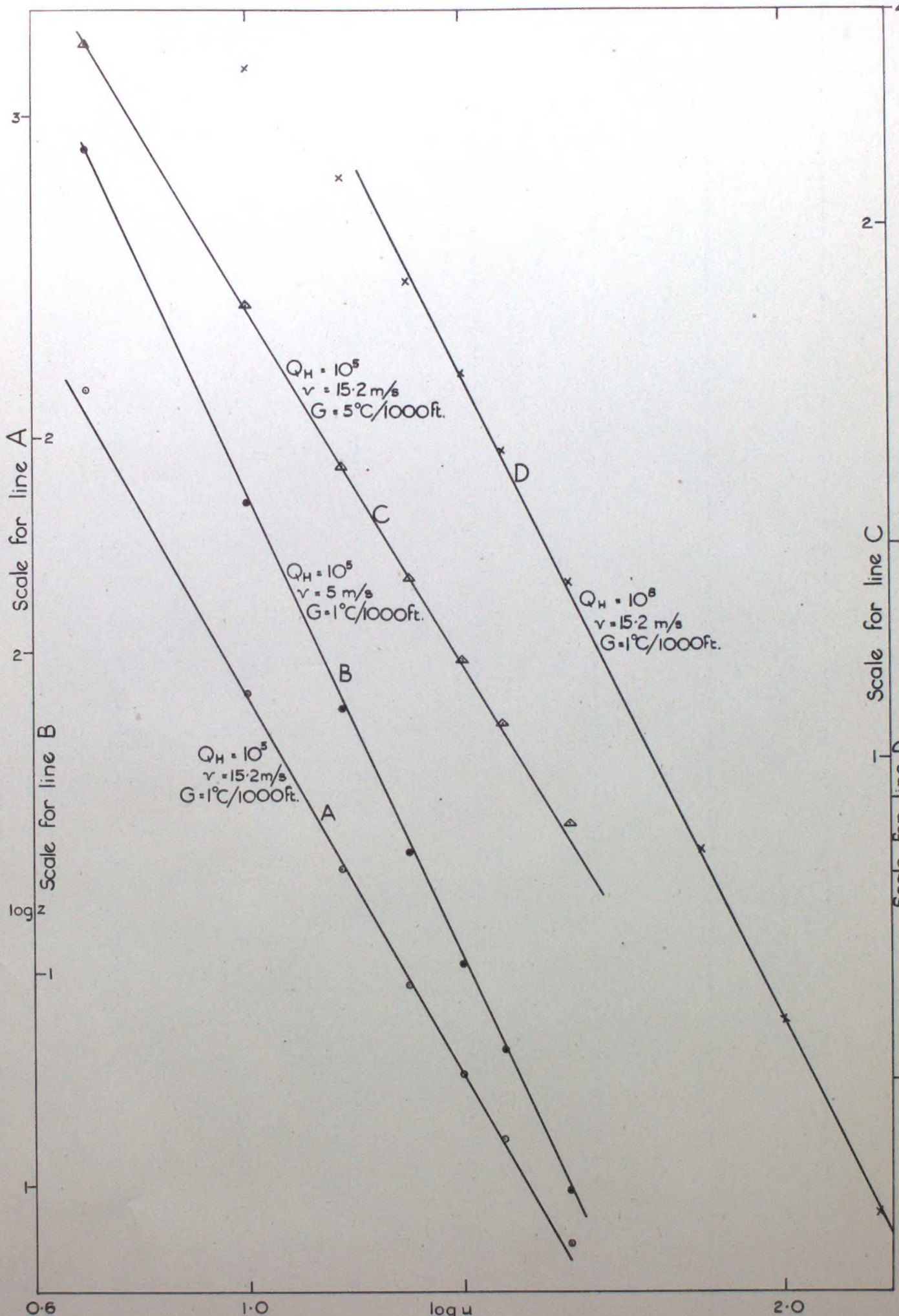

Fig.3. Rise of a plume according to Bosanquet et al as a power of wind speed.